Settings-Source Dependence and Signalling

Guido Bacciagaluppi (Utrecht)

with Ronnie Hermens (Utrecht) and Gijs Leegwater (Rotterdam)

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(or: The strongest yet non-locality theorem)

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QM as a statistical theory (quasi à la von Neumann 1927) :

statistical ensemble(s) ρ

measurement procedures I, J

outcomes o^I, o^J

expectation values $\langle o^I \rangle_\rho$, $\langle o^J \rangle_\rho$

IF QM is complete, then (operationally defined) ensembles corresponding to pure quantum states are statistically *homogeneous*:

If $\quad \rho = \alpha\sigma + (1-\alpha)\tau \quad (0<\alpha<1)$

then $\quad <o^I>_\sigma = <o^I>_\tau \quad$ for all I

If QM is incomplete, there is the possibility of decomposing ensembles by means of additional variables:

$$\rho = \alpha\sigma + (1-\alpha)\tau$$

But it could still be the case that the thus defined ensembles have the same statistical properties as the original ensemble:

$$\langle o^I \rangle_\sigma = \langle o^I \rangle_\tau \qquad \text{for all I}$$

If so, we call such a h.v. theory *trivial*.

Without constraints on h.v. theories, one can easily construct even deterministic ones (in which the hidden variables λ completely fix the outcomes of all measurements), e.g. h.v. are the outcomes of all possible measurements, distributed with the product of the quantum probabilities.

Two famous examples of constraints: expectation values are the same for all measurement procedures corresponding to the same quantum observables; additionally, they are linear with respect to the algebraic structure of the quantum observables.

With these constraints, it follows that ensembles corresponding to pure quantum states are indeed homogeneous (Gleason 1957, von Neumann 1927).

But maybe question-begging (Grete Hermann 1935).

Better: *locality* constraints.

Bipartite measurements: $o^{IJ} = (a^{IJ}, b^{IJ})$

No-signalling: $\langle a^{IJ} \rangle_\rho = \langle a^{IJ'} \rangle_\rho$ and $\langle b^{IJ} \rangle_\rho = \langle b^{I'J} \rangle_\rho$

No-signalling in principle (parameter independence):

$\langle a^{IJ} \rangle_\lambda = \langle a^{IJ'} \rangle_\lambda$ and $\langle b^{IJ} \rangle_\lambda = \langle b^{I'J} \rangle_\lambda$

Theorems based on no-signalling in principle:

(1) Bell (1964): any deterministic h.v. theory has signalling in principle.

(2) Bell (1971): any 'outcome-independent' h.v. theory has signalling in principle.

(3) CRL (Colbeck and Renner 2010, 2011; Leegwater 2016): *any* non-trivial h.v. theory has signalling in principle.

Two standard readings:

- • 'bad' signalling rules out h.v. theories
- • signalling is a striking feature of h.v. theories

N.B. The latter reading (Bell's) suggests a (remote) possibility of investigating h.v. theories experimentally.

But all these theorems assume also *settings-source independence*: in an ensemble ρ corresponding to some quantum state, the distribution ρ(λ) of hidden variables is independent of I,J.

Example: Bell (1964). Since the h.v. theory is deterministic, we have definite outcomes a^{IJ}_λ and b^{IJ}_λ. Now consider

$$a^{IJ}_\lambda b^{IJ}_\lambda + a^{I'J}_\lambda b^{I'J}_\lambda + a^{IJ'}_\lambda b^{IJ'}_\lambda - a^{I'J'}_\lambda b^{I'J'}_\lambda \qquad (*)$$

Assume no-signalling in principle, i.e.

$$a^{I}_\lambda := a^{IJ}_\lambda = a^{IJ'}_\lambda \text{ and } b^{J}_\lambda := b^{IJ}_\lambda = b^{I'J}_\lambda$$

Then (*) becomes

$$a^{I}_\lambda b^{J}_\lambda + a^{I'}_\lambda b^{J}_\lambda + a^{I}_\lambda b^{J'}_\lambda - a^{I'}_\lambda b^{J'}_\lambda$$

Averaging terms separately with $\rho^{IJ}(\lambda)$, $\rho^{I'J}(\lambda)$, $\rho^{IJ'}(\lambda)$, $\rho^{I'J'}(\lambda)$ yields a result in [-4,+4]. Only averaging them simultaneously with $\rho(\lambda)$ yields a result in [-2,+2] that fails to reach the quantum bound.

And similarly for Bell (1971) and for CRL.

But it surely ought to be possible to prove such results even without settings-source independence.

After all, with settings-source dependence, different J, J' mean different $\rho^{IJ}(\lambda)$, $\rho^{IJ'}(\lambda)$, and thus *in general* presumably different probabilities on Alice's side.

Indeed, our theorem is that *any non-trivial h.v. theory with or without settings-source independence leads to signalling*.

Most crucial bit: careful formulation.

In the general case of settings-source dependence, the *same* preparation procedure yields *different* distributions $\rho^{IJ}(\lambda)$ of hidden variables, which *collectively* reproduce the quantum predictions for all I,J.

Now imagine a hypothetical preparation procedure that prepares not only an ensemble ρ that exhibits the statistics predicted by quantum mechanics, but also decomposes it non-trivially into two sub-ensembles: $\rho = \alpha\sigma + (1-\alpha)\tau$.

If we have a h.v. theory with settings-source dependence then for each I,J this same procedure will produce a different distribution $\rho^{IJ}(\lambda)$ and a different pair of sub-distributions $\sigma^{IJ}(\lambda)$ and $\tau^{IJ}(\lambda)$, all of them satisfying

$$\rho^{IJ}(\lambda) = \alpha\sigma^{IJ}(\lambda) + (1-\alpha)\tau^{IJ}(\lambda)$$

And when we write $< a^{IJ} >_\rho$, $< b^{IJ} >_\sigma$, ..., we mean

$$< a^{IJ} >_{\rho IJ(\lambda)} , < b^{IJ} >_{\sigma IJ(\lambda)} , ...$$

[Of course, in the special case in which $\rho^{IJ}(\lambda) = \rho(\lambda)$ etc., the operationally defined ensembles ρ, σ, τ always correspond to the same theoretical distribution $\rho(\lambda)$, $\sigma(\lambda)$, $\tau(\lambda)$.]

The statement of the theorem is now:

For all hypothetical procedures preparing non-trivial non-quantum decompositions of a quantum ensemble

$$\rho = \alpha\sigma + (1-\alpha)\tau \qquad (0<\alpha<1),$$

if σ (and therefore τ) are non-signalling, i.e. if for all I,I',J,J'

$$< a^{IJ} >_\sigma = < a^{IJ'} >_\sigma \quad \text{and} \quad < b^{IJ} >_\sigma = < b^{I'J} >_\sigma,$$

then the h.v. theory is trivial, i.e. for all I,J

$$< a^{IJ} >_\sigma = < a^{IJ} >_\tau \quad \text{and} \quad < b^{IJ} >_\sigma = < b^{IJ} >_\tau.$$

And the proof is also trivial.

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Indeed, (1) we have already implicitly assumed that α is independent of I,J.

(Otherwise, we would have a straightforward case of macroscopic signalling from the settings to the source, and we could relay signals between Alice and Bob faster than light.)

But then, (2) we can formally label σ and τ with the values of a binary hidden variable ξ, whose distribution is now source-independent.

Further, (3) no-signalling for σ and τ means parameter independence for this new hidden variables theory.

But now, (4) by the standard CRL theorem, this new h.v. theory is trivial, which means our original, possibly settings-source-dependent h.v. theory is trivial.

QED

Settings-source dependence is generally neglected, perhaps because it seems crazy.

There are three options:

- hidden variables influence the settings
- settings influence the hidden variables (retrocausality)
- some common cause in the distant past influences both settings and hidden variables (superdeterminism)

The first option is clearly to be ruled out (one can ensure experimentally that something else determines the settings).

But the other two have some definitely non-crazy champions (respectively, Huw Price and Gerard 't Hooft).

And, indeed, standard objections (insistence on 'free will', prejudices against retrocausality) are of a dubious nature.

Superdeterminism may seem conspiratorial, but maybe only because we have not imagined a mechanism yet that makes it look natural.

And retrocausality is the only option that is compatible with any notion of free will one might espouse.

Crucially, both approaches appear to allow for Lorentz-invariant implementation, precisely because they lack action at a distance.

Finally, both approaches provide seemingly unassailable loopholes for the Bell experiments.

Extending the non-locality theorems opens the (very remote) possibility of a direct empirical investigation also of these approaches.

THANK YOU